SOLUSI UJIAN PAI A70

UJIAN A70

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A70-Pemodelan dan Teori Risiko

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Berikut merupakan solusi ujian PAI yang saya buat secara khusus untuk teman-teman PT Padma Radya Aktuaria, dan secara umum untuk teman-teman yang mau mengambil ujian PAI A70. Semoga bermanfaat.

1) Diketahui $(x+1)h'(x) = h(x), x \geq 0$ dan $S(2) = 0{,}5$.
Akan dicari nilai dari $h(0)$.
$(x+1)h'(x) = h(x)$
$$\Leftrightarrow \frac{h'(x)}{h(x)} = \frac{1}{(x+1)}$$
$$\Leftrightarrow d \ln h(x) = \frac{dx}{(x+1)}$$
$$\Leftrightarrow \int_0^x d \ln h(x) = \int_0^x \frac{dx}{(x+1)}$$
$$\Leftrightarrow \ln \frac{h(x)}{h(0)} = \ln(x+1)$$
$$\Leftrightarrow h(x) = (x+1)h(0).$$
Karena $S(x) = \exp\left(-\int_0^x h(x)dx\right)$, maka
$$S(2) = \exp\left(-\int_0^2 h(x)dx\right)$$
$$\Leftrightarrow 0{,}5 = \exp\left(-\int_0^2 (x+1)h(0)dx\right)$$
$$\Leftrightarrow 0{,}5 = \exp(-4h(0))$$
$$\Leftrightarrow -\ln 2 = -4h(0)$$
$$\Leftrightarrow \frac{\ln 2}{4} = h(0).$$
$\therefore$ Jawabannya C.

2) Diketahui X berdistribusi lognormal - μ, σ dengan
$$E(X) = e^3 \Leftrightarrow \exp\left(\mu + \frac{\sigma^2}{2}\right) = e^3$$
$$\Leftrightarrow \mu + \frac{\sigma^2}{2} = 3$$
dan
$$Var(X) = e^{10} - e^6$$
$$\Leftrightarrow E(X^2) - E(X)^2 = e^{10} - e^6$$
$$\Leftrightarrow \exp(2\mu + 2\sigma^2) - e^6 = e^{10} - e^6$$
$$\Leftrightarrow 2\mu + 2\sigma^2 = 10$$
Akan dicari nilai dari $S_X(e^2)$.
Berdasarkan persamaan $\mu + \frac{\sigma^2}{2} = 3$ dan $2\mu + 2\sigma^2 = 10$ didapat $\sigma = 2,\ \mu = 1$.
Karena $F_X(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$, maka
$$S_X(e^2) = 1 - F_X(e^2)$$
$$= 1 - \Phi\left(\frac{\ln e^2 - \mu}{\sigma}\right)$$
$$= 1 - \Phi\left(\frac{2-1}{2}\right)$$
$$= 1 - \Phi\,(0{,}5)$$
$$= 1 - 0{,}6915$$
$$= 0{,}3085$$
$\therefore$ Jawabannya B.

3) Diketahui X bedistribusi gamma - α, θ dengan $\mu = \alpha\theta = 8$, dan
$$\frac{E[(X-\mu)^3]}{\sigma^3} = 1$$
$$\Leftrightarrow E[(X-\mu)^3] = \sigma^3$$
$$\Leftrightarrow E(X^3) - 3\mu E(X^2) + 2\mu^3 = \sigma^3$$
$$\Leftrightarrow \alpha(\alpha+1)(\alpha+2)\theta^3 - 24\alpha(\alpha+1)\theta^2 + 1024 = \sigma^3$$
$$\Leftrightarrow -1024 + 16\theta^2 + 1024 = \sigma^3$$
$$\Leftrightarrow 16\theta^2 = (\alpha\theta^2)^{3/2}$$
$$\Leftrightarrow 16\theta^2 = (8\theta)^{3/2}$$
$$\Leftrightarrow \theta^{1/2} = \frac{8^{3/2}}{16}$$
$$\Leftrightarrow \theta = 2$$
Karena $\alpha\theta = 8$ dan $\theta = 2$, maka $\alpha = 4$.
Sehingga didapat nilai $\sigma^2 = \alpha\theta^2 = 16$
$\therefore$ Jawabannya C.

4) Diketahui $f_X(x) = \frac{e^{-1/x}}{x^2}, x > 0$.
$$F_Y(y) = \Pr(Y \leq y) = \Pr(\theta X \leq y)$$
$$= \Pr\left(X \leq \frac{y}{\theta}\right) = \int_0^{y/\theta} \frac{e^{-1/x}}{x^2} dx$$
$$\Leftrightarrow F_Y(y) = e^{-\theta/y} - 1$$
$$\Leftrightarrow F_Y'(y) = f_Y(y) = \frac{\theta e^{-\theta/y}}{y^2}$$
$\therefore$ Jawabannya B.
Catatan: seharusnya pilihan jawabannya dalam variable y bukan variable x.

5) Diketahui $f_X(x) = \frac{1}{c}, x \in (0, c)$ dan $Y = 2X$.
$$F_Y(y) = \Pr(Y \leq y) = \Pr(2X \leq y)$$
$$= \Pr\left(X \leq \frac{y}{2}\right) = \int_0^{y/2} \frac{1}{c} dx = \frac{y}{2c}$$
Karena $F_Y(y) = y/2c$, maka $f_Y(y) = 1/2c$.

$\therefore$ Jawabannya C.

6) Diketahui

$$f_X(x) = \frac{\tau(x/\theta)^\tau e^{-(x/\theta)^\tau}}{x}, x > 0$$

dan variable acak $Y = g(X)$ dengan pdf

$$f_Y(y) = \frac{e^{-y/\theta^\tau}}{\theta^\tau}, y > 0$$

Akan dicari fungsi $g(X)$. Karena $Y = g(X)$ menyatakan variabel acak Y merupakan fungsi dari variabel acak X, maka haruslah $F_Y(y) = F_X(x)$.

$F_Y(y) = F_X(x)$
$\Leftrightarrow F_Y(g(x)) = F_X(x)$
$\Leftrightarrow f_Y(g(x))\dfrac{dg(x)}{dx} = f_X(x)$
$\Leftrightarrow \dfrac{e^{-g(x)/\theta^\tau}}{\theta^\tau}\dfrac{dg(x)}{dx} = \dfrac{\tau(x/\theta)^\tau e^{-(x/\theta)^\tau}}{x}$
$\Leftrightarrow \dfrac{e^{-g(x)/\theta^\tau}}{\theta^\tau}\dfrac{dg(x)}{dx} = \dfrac{e^{-x^\tau/\theta^\tau}}{\theta^\tau}\tau x^{\tau-1}$
$\Leftrightarrow \dfrac{e^{-g(x)/\theta^\tau}}{\theta^\tau}\dfrac{dg(x)}{dx} = \dfrac{e^{-x^\tau/\theta^\tau}}{\theta^\tau}\dfrac{dx^\tau}{dx}$
$\Leftrightarrow -\dfrac{d\left(e^{-g(x)/\theta^\tau}\right)}{dx} = -\dfrac{d\left(e^{-x^\tau/\theta^\tau}\right)}{dx}$
$\Leftrightarrow e^{-g(x)/\theta^\tau} = e^{-x^\tau/\theta^\tau}$
$\Leftrightarrow g(x) = x^\tau$
$\therefore$ Jawabannya E.

7) Diketahui

$$f(x) = \frac{x(4-x)}{9}, 0 < x < 3$$

Akan dicari nilai $E(X \wedge 1) = \int_0^1 S(x)dx$.

Karena $S(x) = \int_x^3 f(x)dx = \int_x^3 \frac{x(4-x)}{9}dx = 1 - \left(\frac{2x^2 - \frac{x^3}{3}}{9}\right)$

maka

$$E(X \wedge 1) = \int_0^1 1 - \left(\frac{2x^2 - \frac{x^3}{3}}{9}\right)dx$$

$= 0,935$
$\therefore$ Jawabannya D.

8) Diketahui $F_X(x) = 1 - e^{-(x/\theta)^2}$ dan $F_X(100) = 50\%$. Akan dicari nilai dari $\Pr((1+10\%)X \le 100)$.

$F_X(x) = 1 - e^{-(x/\theta)^2}$
$\Leftrightarrow F_X(100) = 1 - e^{-(100/\theta)^2}$
$\Leftrightarrow 0,5 = 1 - e^{-(100/\theta)^2}$
$\Leftrightarrow e^{-(100/\theta)^2} = 0,5$
$\Leftrightarrow (100/\theta)^2 = \ln 2$
$\Leftrightarrow \theta = 100/\sqrt{\ln 2}$

Karena $\theta = 100/\sqrt{\ln 2}$, maka
$\Pr(1,1\,X \le 100) = \Pr(X \le 100/1,1)$
$= 1 - e^{-\left(\frac{100/1,1}{100/\sqrt{\ln 2}}\right)^2}$
$= 1 - e^{-\left(\frac{\sqrt{\ln 2}}{1,1}\right)^2}$
$= 0,436$
$\therefore$ Jawabannya A.

9) Diketahui $f(\alpha) = E(X) - E(X \wedge \alpha)$ dengan $F_X(x) = 1/c, x \in (0,c), c > \alpha$.

Akan dicari nilai $f'(\alpha)$.

$f(\alpha) = E(X) - E(X \wedge \alpha)$
$= \dfrac{c}{2} - \displaystyle\int_0^\alpha \left(1 - \frac{x}{c}\right)dx$
$= \dfrac{c}{2} - \left(\alpha - \dfrac{\alpha^2}{2c}\right)$

Karena $f(\alpha) = \frac{c}{2} - \left(\alpha - \frac{\alpha^2}{2c}\right)$, didapat
$f'(\alpha) = -1 + (\alpha/c)$
$\therefore$ Jawabannya D.

10) Misalkan d=deductible tahun ini dan d'=deductible tahun depan.

Diketahui $f_X(x) = \frac{e^{-x/\theta}}{\theta}, x > 0$
dan $LER = 70\%$ dengan $d' = \frac{4}{3}d$.

$LER = \dfrac{E(X \wedge d)}{E(X)}$
$\Leftrightarrow 0,7 = \dfrac{\int_0^d e^{-x/\theta}dx}{\theta}$
$\Leftrightarrow 0,7 = -e^{-d/\theta} + 1$
$\Leftrightarrow e^{-d/\theta} = 0,3$

Akan dicari nilai $LER' = \frac{E(X \wedge d')}{E(X)}$

$$LER' = \frac{E(X \wedge d')}{E(X)} = -e^{-d'/\theta} + 1$$
$$= -e^{-\frac{4}{3}d/\theta} + 1 = -\left(e^{-d/\theta}\right)^{4/3} + 1$$
$$= -(0{,}3)^{4/3} + 1 = 80\%$$
$\therefore$ Jawabannya C.

11) Diketahui $f_X(x) = \frac{e^{-x/1000}}{1000}, x > 0$ dan deductible $d = 500$.

$$LER = \frac{E(X \wedge 500)}{E(X)} = \frac{\int_0^{500} e^{-x/1000} dx}{1000}$$
$$= \frac{1000 - 1000e^{-500/1000}}{1000}$$
$$= 1 - e^{-0{,}5}$$
Akan dicari nilai d yang membuat LER menjadi dua kali lipat.
$$2(LER) = \frac{E(X \wedge d)}{E(X)}$$
$$\Leftrightarrow 2(1 - e^{-0{,}5}) = 1 - e^{-d/1000}$$
$$\Leftrightarrow e^{-d/1000} = -1 + 2e^{-0{,}5} = 0{,}213$$
$$\Leftrightarrow -d/1000 = \ln 0{,}213$$
$$\Leftrightarrow d = 1546$$
$\therefore$ Jawabannya E.

12) Diketahui fungsi survival pareto $S_X(x) = \left(\frac{\theta}{x+\theta}\right)^\alpha$, $E(X-d)_+ = 1105$ dan $E(X-d|X>d) = 1778$. Padahal $E(X-d|X>d) = E(X-d)_+/S_X(d)$. Sehingga didapat $S_X(d) = 1105/1778$ atau $\left(\frac{\theta}{d+\theta}\right)^\alpha = 1105/1778$. Diketahui juga

$$LER = 0{,}2633$$
$$\Leftrightarrow 1 - \frac{E(X-d)_+}{E(X)} = 0{,}2633$$
$$\Leftrightarrow 1 - \frac{1105}{E(X)} = 0{,}2633$$
$$\Leftrightarrow E(X) = \frac{1105}{1 - 0{,}2633} = 1500$$
$$\Leftrightarrow \frac{\theta}{\alpha - 1} = 1500$$
$$\Leftrightarrow \theta = 1500(\alpha - 1)$$
Karena $E(X \wedge d) = E(X) - E(X-d)_+$ maka
$$E(X \wedge d) = 1500 - 1105$$
$$\Leftrightarrow E(X \wedge d) = 395$$
$$\Leftrightarrow \int_0^d \left(\frac{\theta}{x+\theta}\right)^\alpha dx = 395$$
$$\Leftrightarrow \left(\frac{\theta}{d+\theta}\right)^\alpha \left(\frac{d}{1-\alpha} + \frac{\theta}{1-\alpha}\right) - \frac{\theta}{1-\alpha} = 395$$
$$\Leftrightarrow \frac{1105}{1778}\left(\frac{d}{1-\alpha} - 1500\right) + 1500 = 395$$
$$\Leftrightarrow \frac{d}{\alpha - 1} = 278$$
$$\Leftrightarrow d = 278(\alpha - 1)$$
Dengan menggunakan persamaan $\theta = 1500(\alpha - 1)$ dan $d = 278(\alpha - 1)$ bisa diperoleh nilai α sebagai berikut
$$\left(\frac{\theta}{d+\theta}\right)^\alpha = \frac{1105}{1778}$$
$$\Leftrightarrow \left(\frac{1500(\alpha-1)}{278(\alpha-1) + 1500(\alpha-1)}\right)^\alpha = \frac{1105}{1778}$$
$$\Leftrightarrow \left(\frac{1500}{1778}\right)^\alpha = \frac{1105}{1778}$$
$$\Leftrightarrow \alpha = \frac{\ln\left(\frac{1105}{1778}\right)}{\ln\left(\frac{1500}{1778}\right)} = 2{,}8$$
Sehingga diperoleh $\theta = 2700$ dan $d = 500{,}4$. Selanjutnya akan dicari nilai $E(X-2d)_+/S_X(2d)$.
$$\frac{E(X-2d)_+}{S_X(2d)} = \frac{E(X-1000{,}8)_+}{S_X(1000{,}8)}$$
$$= \frac{\int_{1000,8}^{\infty} \left(\frac{2700}{x+2700}\right)^{2,8} dx}{\left(\frac{2700}{3700,8}\right)^{2,8}}$$
$$= \frac{850{,}38}{\left(\frac{2700}{3700,8}\right)^{2,8}} = 2056$$
$\therefore$ Jawabannya D.

13) Diketahui $f_X(x) = \frac{e^{-x/\theta}}{\theta}, x > 0$, dan $E(X-d)_+ = 75\%\, E(X-d|X>d)$. Padahal $E(X-d|X>d) = E(X-d)_+/S_X(d)$, maka $S_X(d) = 0{,}75$.
$$S_X(d) = 0{,}75$$
$$\Leftrightarrow e^{-d/\theta} = 0{,}75$$
$$\Leftrightarrow d/\theta = -\ln(3/4)$$
Akan dicari nilai $\frac{E(X-2d)_+}{E(X-2d|X>2d)} = S_X(2d)$.
$$S_X(2d) = e^{-2d/\theta} = e^{2\ln(3/4)} = 56{,}25\%$$
$\therefore$ Jawabannya E.

14) Diketahui $F_X(x) = 1 - \left(\frac{5}{x+5}\right)^2$ dan $E(X) = \frac{\theta}{\alpha-1} = 5$. Akan dicari nilai dari $LER = E(Y \wedge 10)/E(Y)$ jika variable acak Y adalah 120% kali variable acak X.

$F_Y(y) = \Pr(Y \le y) = \Pr(1{,}2X \le y)$

$= \Pr\left(X \le \frac{y}{1{,}2}\right) = 1 - \left(\frac{5}{\frac{y}{1{,}2}+5}\right)^2 = 1 - \left(\frac{6}{y+6}\right)^2$

Karena $E(X) = 5$, maka $E(Y) = E(1{,}2X) = 6$.

$LER = \frac{E(Y \wedge 10)}{E(Y)} = \frac{\int_0^{10}[1 - F_Y(y)]\,dy}{6}$

$= \frac{\int_0^{10}\left(\frac{6}{y+6}\right)^2 dy}{6} = \frac{15/4}{6} = 5/8$

$\therefore$ Jawabannya B.

15) Diketahui $f_X(x) = 1/c\,, x \in [0, c], c > 1000$ dan $E(X - 1000|X > 1000) = 500$. Akan di cari nilai c.

$E(X - 1000|X > 1000) = 500$

$\Leftrightarrow \frac{E(X-1000)_+}{S_X(1000)} = 500$

$\Leftrightarrow E(X - 1000)_+ = 500\, S_X(1000)$

$\Leftrightarrow \int_{1000}^{c}\left(1 - \frac{x}{c}\right)dx = 500\left(1 - \frac{1000}{c}\right)$

$\Leftrightarrow \frac{c}{2} - 1000 + \frac{10^6}{2c} = 500 - 500\frac{1000}{c}$

$\Leftrightarrow\ c^2 - 3000c + 2.10^6 = 0$

$\Leftrightarrow\ (c - 1000)(c - 2000) = 0$

$\Leftrightarrow\ c = 1000\ atau\ c = 2000$

Karena syarat $c > 0$, maka yang memenuhi syarat adalah $c = 2000$.

$\therefore$ Jawabannya B.

16) Diketahui $E(Y - x|Y > x) = e^{-x},\ x \ge 0$. Akan dicari fungsi survival $S(x)$.

$e^{-x} = E(Y - x|Y > x) = \int_0^{\infty}\frac{S(x+y)}{S(x)}dy$

$\Leftrightarrow\ S(x)e^{-x} = \int_0^{\infty} S(x+y)dy$

$\Leftrightarrow \frac{d(S(x)e^{-x})}{dx} = \frac{d\left(\int_0^{\infty} S(x+y)dy\right)}{dx}$

$\Leftrightarrow\ S'(x)e^{-x} - S(x)e^{-x} = -\int_0^{\infty} f(x+y)dy$

$\Leftrightarrow\ S'(x)e^{-x} - S(x)e^{-x} = S(x+y)]_0^{\infty}$

$\Leftrightarrow\ \frac{S'(x) - S(x)}{e^x} = -S(x)$

$\Leftrightarrow\ S'(x) = S(x) - S(x)e^x$

$\Leftrightarrow\ \frac{S'(x)}{S(x)} = 1 - e^x$

$\Leftrightarrow\ \frac{d\ln S(x)}{dx} = 1 - e^x$

$\Leftrightarrow \int_0^x d\ln S(x) = \int_0^x (1 - e^x)dx$

$\Leftrightarrow \ln S(x) = x - e^x + 1$

$\Leftrightarrow S(x) = \exp(x - e^x + 1)$

$\therefore$ Jawabannya C.

17) Diketahui $f_X(x) = 1/1000\,, x \in [0{,}1000]$ dan $E(X - d'|X > d') = E(X - d|X > d)/2$. Akan dicari nilai d' yang dinyatakan dalam bentuk d.

$E(X - d|X > d) = \frac{\int_0^x S_X(x)dx}{S_X(d)}$

$= \frac{\int_0^x\left(1 - \frac{x}{1000}\right)dx}{\left(1 - \frac{d}{1000}\right)} = \frac{500 - d + \frac{d^2}{2000}}{\left(1 - \frac{d}{1000}\right)}$

$= \frac{(1000-d)^2}{2(1000-d)} = \frac{(1000-d)}{2}$

Karena $E(X - d'|X > d') = E(X - d|X > d)/2$, maka

$\frac{(1000-d')}{2} = \frac{(1000-d)}{4}$

$\Leftrightarrow 2000 - 2d' = 1000 - d$

$\Leftrightarrow d' = 500 + d/2$

$\therefore$ Jawabannya E.

18) Diketahui

$E(Y_A) = E(X|X > d) = E(X - d + d|X > d)$

$= E(X - d|X > d) + E(d|X > d)$

$= \frac{E(X-d)_+}{S_X(d)} + E(d|X > d)$

$= \frac{E(X) - E(X \wedge d)}{1 - F_X(d)} + d$

dan

$E(Y_B) = \frac{E(X \wedge u) - E(X \wedge d)}{1 - F_X(d)}$

Maka nilai

$$E(Y_A) - E(Y_B) = \frac{E(X) - E(X \wedge u)}{1 - F_X(d)} + d$$

$\therefore$ Jawabannya B.

19) Diketahui $f_X(x) = 1/1000\,, x \in [0,1000]$.
Akan dicari *expected loss prepayment* dengan $r = 5\%, d = 100, u = 500$. Jika Y^P merupakan variable acak klaim dengan $r = 5\%, d = 100, u = 500$, maka

$$E(Y^P) = \frac{(1+r)}{S_X\left(\frac{d}{1+r}\right)}\left(E\left(X \wedge \frac{u}{1+r}\right) - E\left(X \wedge \frac{d}{1+r}\right)\right)$$

$$= \frac{(1{,}05)}{S_X\left(\frac{100}{1{,}05}\right)}\left(E\left(X \wedge \frac{500}{1{,}05}\right) - E\left(X \wedge \frac{100}{1{,}05}\right)\right)$$

$$= \frac{(1{,}05)}{1 - \frac{100/1{,}05}{1000}}\left(\int_{100/1{,}05}^{500/1{,}05} 1 - \frac{x}{100}\,dx\right)$$

$= 316$

$\therefore$ Jawabannya C.

20) Misalkan N merupakan variable acak frekuensi klaim dengan $E(N) = 300$ dan Y merupakan variable acak severity klaim setelah naik 50%. Diketahui bahwa distribusi Y sebagai berikut:

Severity Klaim	Probabilitas
60	25%
120	25%
180	25%
300	25%

Maka
$E(Y) = (60 + 120 + 180 + 300).25\% = 165$
dan $E(Y \wedge 100) = (60 + 100 + 100 + 100).25\% = 90$.
Akan dicari nilai $E[N(Y - 100)_+]$.
$E[N(Y - 100)_+] = E(N).E(Y - 100)_+$
$= E(N).\big(E(Y) - E(Y \wedge 100)\big)$
$= 300.(165 - 90) = 22500$.
$\therefore$ Jawabannya D.

21) Diketahui $E(N) = 20$ dan $S_X(x) = e^{-x/100}$.
Akan dicari nilai $E[N(X - 20)_+]$.
$E[N(X - 20)_+] = E(N).E(X - 20)_+$
$= 20\int_{20}^{\infty} S_X(x)\,dx = 20\int_{20}^{\infty} e^{-x/100}\,dx$
$= 20.100.e^{-20/100} = 1637$
$\therefore$ Jawabannya A.

22) Diketahui $E(N) = r\beta = 6$ dan $S_X(x) = \left(\frac{200}{x+200}\right)^3$. Akan dicari nilai $E[N(X - 100)_+]$.
$E[N(X - 100)_+] = E(N).E(X - 100)_+$
$= 6\int_{100}^{\infty} S_X(x)\,dx = 6\int_{100}^{\infty}\left(\frac{200}{x+200}\right)^3 dx$
$= 267$
$\therefore$ Jawabannya C.

23) Diketahui data klaim sebagai berikut:
100; 100; 100; 200; 300; 300; 300; 400; 500; 600. Data tersebut kebetulan sudah urut, sehingga bisa langsung digunakan untuk mencari $S(300)$.

$$S(300) = \frac{banyaknya\ data\ setelah\ 300}{banyaknya\ data\ keseluruhan}$$

$$= \frac{3}{10} = 0{,}3$$

Karena $H(x) = -\ln S(x)$, maka
$H(x) = -\ln S(x) = -\ln 0{,}3 = 1{,}204$.
$\therefore$ Jawabannya D.

24) Data dari soal dapat dinyatakan dalam bentuk tabel berikut:

waktu meninggal	waktu keluar
1	-
-	1
-	3
4	-
5	-
-	6
8	-

Berdasarkan tabel diatas didapat nilai-nilai $S(t)$ sebagai berikut:

waktu meninggal	banyaknya data	urutan ke- (dari belakang)	$S(t)$
1	1	7	$\left(1 - \frac{1}{7}\right) = \frac{6}{7}$
4	1	4	$\frac{6}{7}\left(1 - \frac{1}{4}\right) = \frac{9}{14}$
5	1	3	$\frac{9}{14}\left(1 - \frac{1}{3}\right) = \frac{3}{7}$
8	1	1	$\frac{3}{7}\left(1 - \frac{1}{1}\right) = 0$

Mengacu pada table diatas, didapat $E[S(t)]$ yaitu:

$E[S(t)] = 1.S(1) + 4.S(4) + 5.S(5) + 8.0$

$= 1.\frac{6}{7} + 4.\frac{9}{14} + 5.\frac{3}{7} + 0 = \frac{39}{7} = 5{,}57$

$\therefore$ Jawabannya D.

25) Diketahui $\bar{X} = \frac{130+20+350+218+1822}{5} = 508$ dan

$$\sum_{i=1}^{5} \frac{x_i^2}{5} = \frac{130^2 + 20^2 + 350^2 + 218^2 + 1822^2}{5} = 7014016$$

Karena variable acak X berdistribusi pareto, maka dengan metode moment didapat persamaan $\frac{\theta}{\alpha-1} = E(X) = \bar{X} = 508$ dan $\frac{2\theta^2}{(\alpha-1)(\alpha-2)} = E(X^2) = \sum_{i=1}^{5}\frac{x_i^2}{5} = 7014016$. Akan dicari nilai $E(X \wedge 500)$.

$$\frac{\left(\frac{\theta}{\alpha-1}\right)^2}{\frac{2\theta^2}{(\alpha-1)(\alpha-2)}} = \frac{508^2}{7014016}$$

$$\Leftrightarrow \frac{\left(\frac{\theta}{\alpha-1}\right)^2}{\frac{2\theta^2}{(\alpha-1)(\alpha-2)}} = \frac{508^2}{7014016}$$

$$\Leftrightarrow \frac{(\alpha-2)}{2(\alpha-1)} = \frac{508^2}{7014016}$$

$$\Leftrightarrow 1 - \frac{1}{(\alpha-1)} = \frac{2\,.508^2}{7014016}$$

$$\Leftrightarrow (\alpha-1) = \frac{7014016}{6497888}$$

Pada bagian lampiran yang ada pada soal, rumus $E(X \wedge 500) = \frac{\theta}{\alpha-1}\left(1 - \left(\frac{\theta}{500+\theta}\right)^{\alpha-1}\right)$.

Sehingga

$$E(X \wedge 500) = \frac{\theta}{\alpha-1}\left(1 - \left(\frac{\left(\frac{\theta}{\alpha-1}\right)(\alpha-1)}{500+\left(\frac{\theta}{\alpha-1}\right)(\alpha-1)}\right)^{\alpha-1}\right)$$

$$= 508\left(1 - \left(\frac{(508)\left(\frac{7014016}{6497888}\right)}{500+(508)\left(\frac{7014016}{6497888}\right)}\right)^{\left(\frac{7014016}{6497888}\right)}\right)$$

$= 256$

$\therefore \mathit{Tidak\ ada\ jawaban\ yang\ memenuhi}.$

26) Diketahui $f(x;t) = 2(t-1)t^x$. Akan dicari maksimum likelihood estimator untuk t.

$$L(t) = \prod_{i=1}^{5} 2(t-1)t^{x_i} = 2^5(t-1)^5 t^{\sum_{i=1}^{5} x_i}$$

$$\Leftrightarrow \ln L(t) = 5\ln 2 + 5\ln(t-1) + \textstyle\sum_{i=1}^{5} x_i \ln t$$

$$\Leftrightarrow \frac{d\ln L(t)}{dt} = \frac{5}{(t-1)} + \frac{\sum_{i=1}^{5} x_i}{t}$$

$$\Leftrightarrow 0 = \frac{5}{-(1-t)} + \frac{\sum_{i=1}^{5} x_i}{t}$$

$$\Leftrightarrow \frac{t}{(1-t)} = \frac{\sum_{i=1}^{5} x_i}{5}$$

$$\Leftrightarrow \frac{t}{(1-t)} = \bar{x}$$

$$\Leftrightarrow t = \frac{\bar{x}}{1+\bar{x}}$$

$\therefore$ Jawabannya E.

27) Diketahui $f(x;\sigma) = \frac{\sigma}{2}e^{-\sigma|x-\mu|}$. Akan dicari maksimum likelihood estimator untuk σ.

$$L(\sigma) = \prod_{i=1}^{n} \frac{\sigma}{2} e^{-\sigma|x_i-\mu|} = \left(\frac{\sigma}{2}\right)^n e^{-\sigma\sum_{i=1}^{n}|x_i-\mu|}$$

$$\Leftrightarrow \ln L(\sigma) = \mathrm{n}\ln\left(\frac{\sigma}{2}\right) + -\sigma\sum_{i=1}^{n}|x_i-\mu|$$

$$\Leftrightarrow \frac{d\ln L(\sigma)}{d\sigma} = \frac{n}{\sigma} - \sum_{i=1}^{n}|x_i-\mu|$$

$$\Leftrightarrow 0 = \frac{n}{\sigma} - \sum_{i=1}^{n}|x_i-\mu|$$

$$\Leftrightarrow \sigma = \frac{n}{\sum_{i=1}^{n}|x_i-\mu|}$$

$\therefore$ Jawabannya E.

28) Diketahui variable acak S berdistribusi *Compound Poisson* dan Y merupakan variable acak severity yang berdistribusi konstan. Sedangkan ukuran sampel yang diperlukan untuk full kredibilitas adalah $n = 2670$. Karena Y berdistribusi konstan maka $Var(Y) = 0$. Jika N merupakan variable acak poisson dari *Compound Poisson* S, maka $E(N) = Var(N) = \lambda$. Sementara akan dicari standard untuk full kredibilitas yaitu λ_0.

$$n = \lambda_0 \frac{Var(S)}{E(Y).E(S)}$$

$$\Leftrightarrow 2670 = \lambda_0 \frac{Var(N).E(Y)^2 + E(N).Var(Y)}{E(Y).E(N).E(Y)}$$

$$\Leftrightarrow 2670 = \lambda_0 \frac{\lambda.E(Y)^2 + \lambda.0}{\lambda.E(Y)^2}$$

$\Leftrightarrow 2670 = \lambda_0$.
Jika Y berdistribusi lognormal dengan $E(Y) = 1000, Var(Y) = 1500000$, akan dicari nilai n.

$$n = \lambda_0 \frac{Var(N).E(Y)^2 + E(N).Var(Y)}{E(Y).E(N).E(Y)}$$
$$= \lambda_0 \frac{\lambda(E(Y)^2 + Var(Y))}{\lambda.E(Y)^2}$$
$$= 2670 \frac{(1000^2 + 1500000)}{1000^2} = 6675$$

$\therefore$ Jawabannya B.

29) Diketahui $f(x|\theta) = \theta^2 x \exp(-\theta x)$, $x > 0$.
Akan dicari nilai $E(X|\theta)$.

$$E(X|\theta) = \int_0^\infty x\, f(x|\theta) dx$$
$$= \int_0^\infty x^2\theta^2 \exp(-\theta x)\, dx$$
$$= \theta^2 \left(-\frac{x^2}{\theta} - \frac{2x}{\theta^2} - \frac{2}{\theta^3}\right) \exp(-\theta x)\Big]_0^\infty$$
$$= 0 - \theta^2 \left(0 - 0 - \frac{2}{\theta^3}\right) \exp(0) = \frac{2}{\theta}$$

$\therefore$ Jawabannya B.

30) Diketahui

$$f(x,\theta) = f(x|\theta).\pi(\theta)$$
$$= \theta^2 x \exp(-\theta x) . \theta \exp(-\theta)$$
$$= \theta^3 x \exp(-\theta(x+1))$$

Akan dicari nilai $E(X)$.

$$f(x) = \int_0^\infty f(x,\theta) d\theta$$
$$= \int_0^\infty \theta^3 x \exp(-\theta(x+1))\, d\theta = \frac{6x}{(x+1)^4}$$

Karena sudah diketahui fungsi $f(x)$, maka bisa didapat nilai $E(X)$ sebagai berikut:

$$E(X) = \int_0^\infty x f(x) dx$$
$$= \int_0^\infty x \frac{6x}{(x+1)^4} dx = \int_0^\infty \frac{6x^2}{(x+1)^4} dx$$
$$= -\frac{6}{(x+1)} + \frac{6}{(x+1)^2} - \frac{2}{(x+1)^3}\Big]_0^\infty = 2$$

$\therefore$ Jawabannya D.